



# University of Nairobi

## **CLOSURE OF THE UNIVERSE BY WEAKLY INTERACTING MASSIVE NEUTRINOS**

By

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## DECLARATION AND CERTIFICATION

This thesis is an original work submitted for the Degree of Doctor of Philosophy in Physics at the University of Nairobi and has not been, nor will it be presented to any other university for a similar or other degree award.

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MR GEORGE MAUMBA OKEYO

The undersigned certify that he has read and hereby recommend for acceptance by the University of Nairobi a thesis entitled: “Closure of the Universe by Weakly Interacting Massive Neutrinos” in fulfillment of the requirements for the award of the degree of Doctor of Philosophy (Physics) of the University of Nairobi.

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## **DEDICATION**

*To my Parents-Francis Nyaga Okeyo Maumba and Priskah Kerubo, my children Isaac Mecheo, Jacinta Nyamoita, Emmanuel Ondieki and spouse Catherine Mukolwe.*

## ABSTRACT

The role of neutrino mass in the context of grand unified theory has been investigated using the equilibrium condition between a critically expanding universe and weak neutrino interaction rates in the early universe. The critical expansion rate has been studied using the Friedman equation that was modified by expressing the scale factor as a function of time. This gave rise to the model equation used in the investigation to establish the critical nature of the expanding universe. The delicate equilibrium balance between expansion and weak neutrino reaction rates generated the Boltzmann transport equation; this was solved by Successive Approximation technique and a mass value of  $1.57 \text{ GeV}$  was established for heavy or non-relativistic neutrino. For a light or relativistic neutrino, a mass value of  $1.97 \text{ eV}$  was determined. The results were found to generate a cosmological mass gap for neutrino masses in the range between  $1.97 \text{ eV}$  and  $1.57 \text{ GeV}$  as opposed to the standard electroweak model of particle physics that allows existence of a massless neutrino. More importantly, the presence of a  $\text{GeV}$  neutrino mass predicts existence of a fourth family of leptons. When the calculated mass was used in the calculated See-Saw relation, a unification scale of  $1.542 \times 10^{13} \text{ GeV}$  was achieved. This energy scale is interesting since the weak, electromagnetic and strong interactions would all have the same strength at around  $10^{13} \text{ GeV}$ - $10^{16} \text{ GeV}$  which suggests a very similar value for the grand unification scale. This appears promising as the anticipated discovery of tiny neutrino masses may help in probing the structure of the particle physics models that lie beyond the standard electroweak model.

Neutrino mass generation has also been investigated using the standard Higgs mechanism technique. The standard electroweak Lagrangian was modified by adding a mass term to it and a perturbation to the vacuum expectation value generated neutrino mass term couplings. This was found to be true with the aid of scalar fields, which, in this case was the Higgs boson. Also from the theory of neutrino oscillations, it was found that neutrinos of different masses travel with different velocities rather than with the same velocity. In particular, the dynamical effect i.e. the probability of neutrino oscillation was found to be very dependent on neutrino mass. This was studied by using the simple approach of mathematical theory of matrices rather than the complicated methods of renormalization. The results anticipate that the oscillation experiments (such as Fermilab's LBNE and the MINOS) may play an important role in putting the evidence for neutrino mass on a more solid and practical evidence.

## LIST OF SYMBOLS

$L$	Lepton Number
$GUT$	Grand Unified Theories
$B-L$	Baryon-minus-Lepton Number
$SU(2)$	Special Unitary matrix
$U(1)$	Unitary matrix
$GeV$	Giga-electron-volt
$CERN$	European Organization for Nuclear Research
$W^\pm$	Charged weak vector bosons
$Z^0$	Neutral weak vector boson
$Kpc$	Kilo parsec
$M(R)$	Mass within radius R
$He$	Helium
$D$	Deuterium
$Li$	Lithium
$\Omega_B$	Relative energy density of Baryons
$\dots_\gamma$	Photon energy density
$\dots_\nu$	Neutrino energy density
$eV$	electron-volt
$TOE$	Theory of Everything
$\beta$	Beta symbol
$KeV$	Kilo-electron-volt
$Q$	Energy release in fission or fusion